C1 MAY 2013 INTERNATIONAL PAPER.

1. Given $y = x^3 + 4x + 1$, find the value of $\dfrac{dy}{dx}$ when $x = 3$

(4)

$y' = 3x^2 + 4 \qquad x = 3 \Rightarrow y' = 27 + 4 = 31$

2. Express $\dfrac{15}{\sqrt{3}} - \sqrt{27}$ in the form $k\sqrt{3}$, where k is an integer.

$\dfrac{15\sqrt{3}}{3} - \sqrt{9}\sqrt{3} = 5\sqrt{3} - 3\sqrt{3} = 2\sqrt{3}$

3. Find

$$\int \left(3x^2 - \frac{4}{x^2}\right) \mathrm{d}x$$

giving each term in its simplest form.

$\int 3x^2 - 4x^{-2}\, dx = x^3 + 4x^{-1} + c$

4. The line L_1 has equation $4x + 2y - 3 = 0$

(a) Find the gradient of L_1.

(2)

The line L_2 is perpendicular to L_1 and passes through the point (2, 5).

(b) Find the equation of L_2 in the form $y = mx + c$, where m and c are constants.

(3)

a) $2y = 3 - 4x \quad \therefore \quad y = -2x + 1.5 \quad \Rightarrow M = -2$

b) perp $\Rightarrow M = \frac{1}{2} \quad L_2 \Rightarrow y - 5 = \frac{1}{2}(x - 2)$

$\Rightarrow 2y - 10 = x - 2 \quad \Rightarrow x - 2y + 8 = 0$

5. Solve

(a) $2^y = 8$

(1)

(b) $2^x \times 4^{x+1} = 8$

a) $y=3$

b) $2^x \times (2^2)^{x+1} = 2^3$

$2^x \times 2^{2x+2} = 2^3$

$2^{3x+2} = 2^3$ $\therefore 3x+2=3$

$3x = 1$

$x = \frac{1}{3}$

6. A sequence $x_1, x_2, x_3 \ldots$ is defined by

$$x_1 = 1$$

$$x_{n+1} = (x_n)^2 - kx_n, \quad n \geqslant 1$$

where k is a constant, $k \neq 0$

(a) Find an expression for x_2 in terms of k. **(1)**

(b) Show that $x_3 = 1 - 3k + 2k^2$ **(2)**

Given also that $x_3 = 1$,

(c) calculate the value of k. **(3)**

(d) Hence find the value of $\sum_{n=1}^{100} x_n$ **(3)**

a) $x_1 = 1$

$x_2 = 1^2 - k(1) = 1-k$

b) $x_3 = (1-k)^2 - k(1-k) = 1-2k+k^2-k+k^2$

$= 2k^2 - 3k + 1$ #

c) $2k^2 - 3k + 1 = 1 \Rightarrow 2k^2 - 3k = 0 \Rightarrow k(2k-3) = 0$

$\therefore \underline{k = 1.5}$

d) $x_1 = 1 \quad x_2 = 0.5 \quad x_3 = 1 \quad x_4 = 0.5 \ldots$

$S_{100} = 1 + 0.5 + 1 + 0.5 \ldots$

$= 50(1+0.5) = 75$

7. Each year, Abbie pays into a savings scheme. In the first year she p… £500 … payments then increase by £200 each year so that she pays £700 in the se… in the third year and so on.

(a) Find out how much Abbie pays into the savings scheme in the tenth year.

Abbie pays into the scheme for n years until she has paid in a total of £67 200.

(b) Show that $n^2 + 4n - 24 \times 28 = 0$

(5)

(c) Hence find the number of years that Abbie pays into the savings scheme.

(2)

$u_1 = 500$ $\quad a = 500 \quad d = 200$

$u_2 = 700$

$u_{10} = a + 9d = 500 + 9 \times 200 = £2300$

b) $S_n = \frac{1}{2}n\left(2a + (n-1)d\right) \Rightarrow \frac{1}{2}n\left(1000 + (n-1)200\right) = 67200$

$\Rightarrow 134400 = n\left(1000 + 200n - 200\right)$

$\Rightarrow 1344\cancel{00} = 2\cancel{00}n^2 + 8\cancel{00}n \Rightarrow 2n^2 + 8n - 1344 = 0$

$\Rightarrow n^2 + 4n - 24 \times 28 = 0$ #

c) $(n-24)(n+28) = 0 \quad \therefore n = 24$

8. A rectangular room has a width of x m.

The length of the room is 4 m longer than its width.

Given that the perimeter of the room is greater than 19.2 m,

(a) show that $x > 2.8$

(3)

Given also that the area of the room is less than 21 m^2,

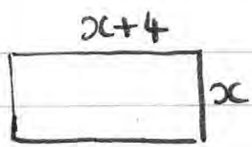
(b) (i) write down an inequality, in terms of x, for the area of the room.

(ii) Solve this inequality.

(4)

(c) Hence find the range of possible values for x.

(1)

a) 

$P = 2(x+x+4) = 4x+8$

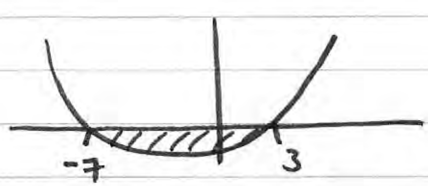
$4x+8 > 19.2 \Rightarrow 4x > 11.2$

$x > 2.8$ #

b) area $= x(x+4) < 21 \Rightarrow x^2+4x-21 < 0$

$(x+7)(x-3) < 0$

$x = -7 \quad x = 3$



$\therefore \quad -7 < x < 3$

c) $0 < x < 3$

9.

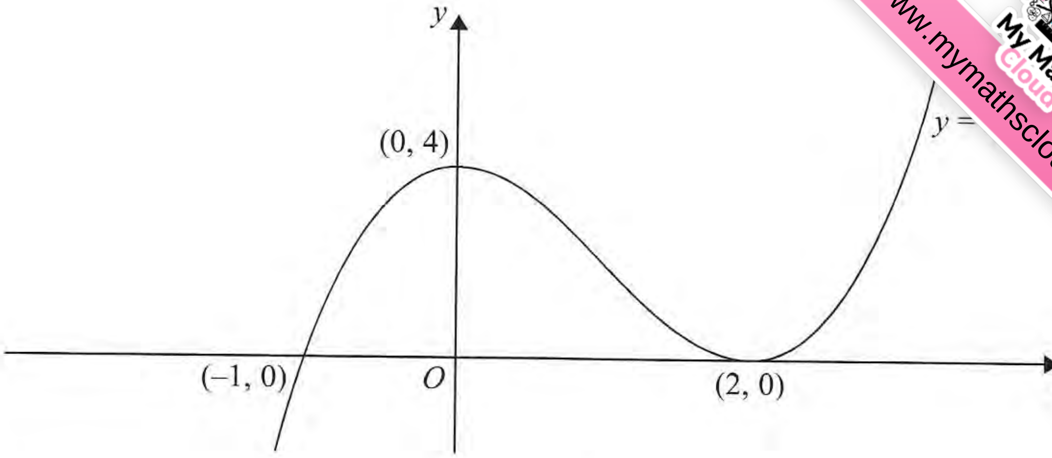


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.

The curve C passes through the point $(-1, 0)$ and touches the x-axis at the point $(2, 0)$.

The curve C has a maximum at the point $(0, 4)$.

(a) The equation of the curve C can be written in the form

$$y = x^3 + ax^2 + bx + c$$

where a, b and c are integers.

Calculate the values of a, b and c.

(5)

(b) Sketch the curve with equation $y = \mathrm{f}(\tfrac{1}{2}x)$ in the space provided on page 24

Show clearly the coordinates of all the points where the curve crosses or meets the coordinate axes.

(3)

a) $y = (x+1)(x-2)^2 = (x+1)(x^2-4x+4)$

$y = x^3 - 4x^2 + 4x$

$\quad + x^2 - 4x + 4$

$y = x^3 - 3x^2 + 4$ $\quad a = -3 \quad b = 0 \quad c = 4$

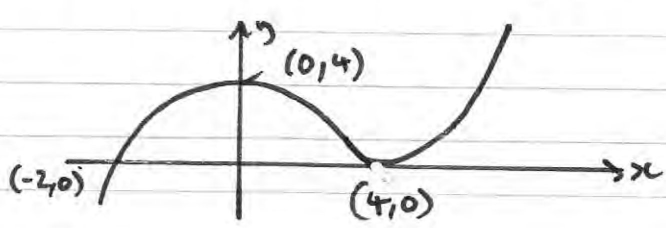
b) $f(\tfrac{1}{2}x)$

$\rightarrow \tfrac{1}{2} \leftarrow$

$=$

$\leftarrow 2 \rightarrow$

$\times x$ by 2



10. A curve has equation $y = f(x)$. The point P with coordinates $(9, 0)$ lies on the curve.

Given that

$$f'(x) = \frac{x + 9}{\sqrt{x}}, \qquad x > 0$$

(a) find $f(x)$.

(6)

(b) Find the x-coordinates of the two points on $y = f(x)$ where the gradient of the curve is equal to 10

(4)

a) $f'(x) = \frac{x^1}{x^{\frac{1}{2}}} + \frac{9}{x^{\frac{1}{2}}} = x^{\frac{1}{2}} + 9x^{-\frac{1}{2}}$

$f(x) = \frac{2}{3}x^{\frac{3}{2}} + 18x^{\frac{1}{2}} + C$

b) $\frac{x+9}{\sqrt{x}} = 10 \quad \Rightarrow \quad x + 9 = 10\sqrt{x}$

$\Rightarrow \quad x^2 + 18x + 81 = 100x$

$\Rightarrow \quad x^2 - 82x + 81 = 0$

$\Rightarrow \quad (x - 81)(x - 1) = 0$

$x = 1, \; x = 81$

11.

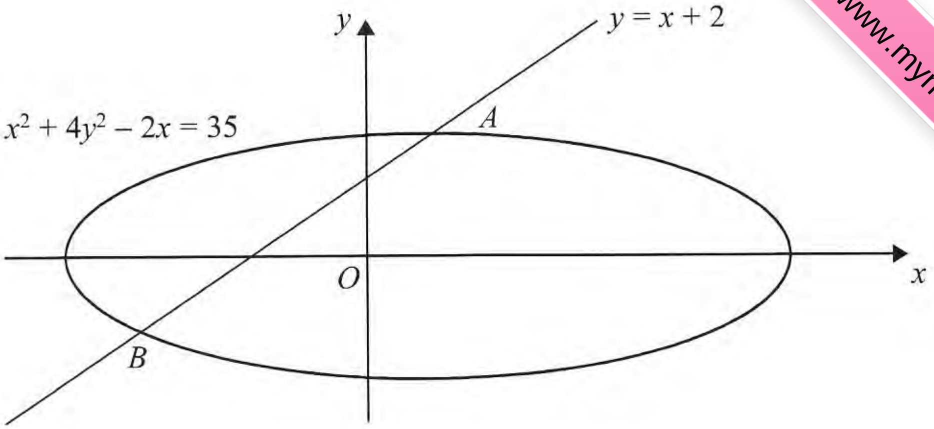


Figure 2

The line $y = x + 2$ meets the curve $x^2 + 4y^2 - 2x = 35$ at the points A and B as shown in Figure 2.

(a) Find the coordinates of A and the coordinates of B.

(6)

(b) Find the distance AB in the form $r\sqrt{2}$ where r is a rational number.

(3)

$y^2 = x^2 + 4x + 4$ $\quad\quad x^2 + 4x^2 + 16x + 16 - 2x - 35 = 0$

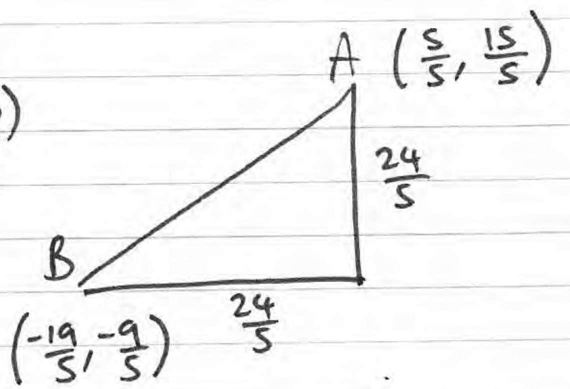
$\Rightarrow 5x^2 + 14x - 19 = 0$

$\Rightarrow (5x + 19)(x - 1) = 0$

$\Rightarrow x = \frac{-19}{5}$ | $x = 1$ $\quad (1, 3)$

$y = \frac{-9}{5}$ | $y = 3$ $\quad \left(\frac{-19}{5}, \frac{-9}{5}\right)$

b)



$AB^2 = \left(\frac{24}{5}\right)^2 + \left(\frac{24}{5}\right)^2$

$AB^2 = 2 \times \left(\frac{24}{5}\right)^2$

$\therefore AB = \frac{24}{5}\sqrt{2}$